

Grade: 6

Enduring Skill 1:

Develop an understanding to connect ratio and rate to whole number multiplication and division, and use concepts of ratio and rate to solve problems

Demonstrators and Related Standards:

1. Use reasoning about multiplication and division to solve ratio and rate problems about quantities (6.RP.3b, 6.RP.3c, 6.RP.3d)
2. Connect understanding of multiplication and division with ratios and rates (6.RP.2, 6.RP.3a)
3. Connect ratios and fractions using multiplication and division (6.RP.1)
4. Solve wide variety of problems involving ratios and rates (6.RP.3, 6.EE.6, 6.EE.9)

Assessment Items:

1. ES 1, Demonstrator 2, Standard (6.RP.2)

The grocery store sells beans in bulk. The grocer's sign above the beans says,
5 pounds for \$4.

At this store, you can buy any number of pounds of beans at this same rate, and all prices include tax.

Alberto said, "*The ratio of the number of dollars to the number of pounds is 4:5. That's \$0.80 per pound.*"

Beth said, "*The sign says the ratio of the number of pounds to the number of dollars is 5:4. That's 1.25 pounds per dollar.*"

- a) Are Alberto and Beth both correct? Explain.
- b) Claude needs two pounds of beans to make soup. How much money will he need? Explain and/or show your work.
- c) Donna has \$10 and wants to stock up on beans. How many pounds of beans she can buy? Explain and/or show your work.
- d) Do you prefer to answer parts (b) and (c) using Alberto's rate of \$0.80 per pound, using Beth's rate of 1.25 pounds per dollar, or use different strategies for each part? Explain.

2. ES 1, Demonstrator 2, Standard (6.RP.2)

Tyler can swim 5 laps of the school pool in 3 minutes, Jose can swim 6 laps in 4 minutes, and Kevin can swim 4 laps in 2 minutes. Who is the fastest swimmer? Explain and show your work.

3. ES 1, Demonstrators 2 and 4, Standard (6.RP.3, 3a)

A mixture of concrete is made up of sand and cement in a ratio of 5 : 3. How many cubic feet of each are needed to make 160 cubic feet of concrete mix? Complete the following table and explain how you found your answers.

Sand (cubic feet)	5	10	
Cement (cubic feet)	3	6	
Total (cubic feet)		16	160

4. ES 1, Demonstrator 1 and 4, Standard (6.RP.3, 3b)

A board was cut into two pieces whose lengths are in the ratio 2:5. The longest piece was 85 inches. How long was the shortest piece? Show your work.

5. ES 1, Demonstrator 1 and 4 , Standard (6.RP.3, 3b)

At Al's Catering Service, they often make a large batch of fruit salad. The ratio of cups of apples to cups of oranges in the recipe is 8 : 5.

- a) If they use 15 cups of oranges, how many cups of apples should they use?
- b) If they only have 1 cup of oranges, how many cups of apples should they use?

6. ES 1, Demonstrators 1,2 and 4, Standard (6.RP.3)

At a track meet, $\frac{5}{8}$ of the students were boys. There were 40 boys. How many girls attended the track meet? Show your work.

7. ES 1, Demonstrator 1, Standard (6.RP.3)

Fresh Market sells three sizes of bags of pistachios. The big bag is 20 ounces and costs \$8.00. The medium bag is 16 ounces and costs \$7.20. The small bag is 12 ounces and costs \$6.00.

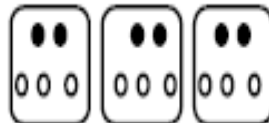
- a) What is the cost per ounce for each bag size? Show your work.
- b) What size is the best buy? Explain your reasoning.

8. ES 1, Demonstrator 3, Standard (6.RP.1)

A comparison of 6 guppies and 9 goldfish could be expressed in any of the following forms: $\frac{6}{9}$, 6 to 9 or 6:9. If the number of guppies is represented by black circles and the number of goldfish is represented by white circles, this ratio could be modeled as



These values can be regrouped into 2 black circles (goldfish) to 3 white circles (guppies), which would reduce the ratio to, $\frac{2}{3}$, 2 to 3 or 2:3.

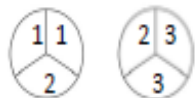


9. ES 1, Demonstrator 2, Standard (6.RP.2)

There are 2 cookies for 3 students. What is the amount of cookie each student would receive? (i.e. the unit rate)

Solution: This can be modeled as shown below to show that there is $\frac{2}{3}$ of a cookie for 1 student, so the unit rate is

$\frac{2}{3}$: 1.

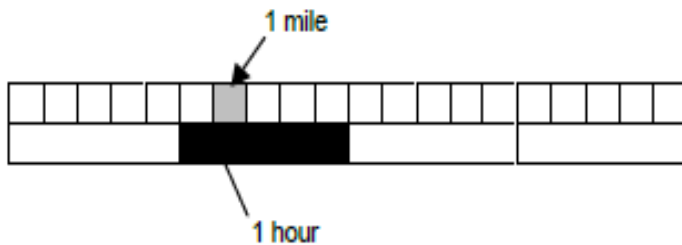


10. ES 1, Demonstrator 2, Standard (6.RP.2)

On a bicycle Jack can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance Jack can travel in 1 hour and the amount of time required to travel 1 mile)?

Solution: Jack can travel 5 miles in 1 hour written as $\frac{5 \text{ mi}}{1 \text{ hr}}$ and it takes $\frac{1}{5}$ of a hour to travel each mile written as

$\frac{1 \text{ hr}}{5}$. Students can represent the relationship between 20 miles and 4 hours.



11. ES 1, Demonstrator 1, 2, Standard (6.RP.3)

At Books Unlimited, 4 paperback books cost \$18.

- What would 7 books cost?
- How many books could be purchased with \$54.

12. ES 1, Demonstrator 1, Standard (6.RP.3)

Ratios can also be used in problem solving by thinking about the total amount for each ratio unit. The ratio of cups of orange juice concentrate to cups of water in punch is 1 : 3. If James made 32 cups of punch, how many cups of orange did he need.

13. ES 1, Demonstrator 1, Standard (6.RP.3)

Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

14. ES 1, Demonstrator 4, Standard (6.EE.6)

- Write an expression to represent Susan’s age in three years, when a represents her present age.
- Write an expression to represent the number of wheels, w , on any number of bicycles.
- Write an expression to represent the value of any number of quarters, q .

15. ES 1, Demonstrator 4, Standard (6.EE.9)

What is the relationship between the two variables?
Write an expression that illustrates the relationship.

x	1	2	3	4
y	2.5	5	7.5	10

16. ES 1, Demonstrator 3, Standard (6.RP.1)

The students in Mr. Hill’s class played games at recess.

- 6 boys played soccer
- 4 girls played soccer
- 2 boys jumped rope
- 8 girls jumped rope

Afterward, Mr. Hill asked the students to compare the boys and girls playing different games. Mika said, “Four more girls jumped rope than played soccer.” Chaska said, “For every girl that played soccer, two girls jumped rope.” Mr. Hill said, “Mika compared the girls by looking at the difference and Chaska compared the girls using a ratio.”

- Compare the number of boys who played soccer and jumped rope using the difference. Write your answer as a sentence as Mika did.
- Compare the number of boys who played soccer and jumped rope using a ratio. Write your answer as a sentence as Chaska did.
- Compare the number of girls who played soccer to the number of boys who played soccer using a ratio. Write your answer as a sentence as Chaska did.

Solutions

Solution: Answers

1. Four more boys played soccer than jumped rope.
2. For every three boys that played soccer, one boy jumped rope. Therefore the ratio of the number of boys that played soccer to the number of boys that jumped rope is 3:1 (or "three to one").
3. For every two girls that played soccer, three boys played soccer. Therefore the ratio of the number of girls that played soccer to the number of boys that played soccer is 2:3 (or "two to three").

17. ES 1, Demonstrator 3, Standard (6.RP.1)

In a bag of marbles, $\frac{3}{5}$ of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?

solution: $\frac{4}{7}$

18. ES 1, Demonstrator 3, Standard (6.RP.1)

Ty took the escalator to the second floor. The escalator is 12 meters long, and he rode the escalator for 30 seconds. Which statements are true? Select all that apply.

- a. He traveled 2 meters every 5 seconds.
- b. Every 10 seconds he traveled 4 meters.
- c. He traveled 2.5 meters per second.
- d. He traveled 0.4 meters per second.
- e. Every 25 seconds, he traveled 7 meters.

solution: a, b, d are all correct

Grade 6

Enduring Skill 2:

Understand division of fractions and extend ideas of numbers to the system of rational numbers, which includes negative numbers

Demonstrators and Related Standards:

1. Uses the meaning of fractions, multiplication, and division, and the relationship between multiplication and division to understand and explain why procedures of dividing fractions make sense (6.NS.1)
2. Uses multiplication and division to solve problems (6.NS.2, 6.NS.3, 6.NS.4)
3. Extend the previous understanding of number and the ordering of numbers to the full system of rational numbers, and in particular negative integers (6.NS.6a, 6.NS.5)
4. Reason about the order and absolute value of rational numbers, and about the location of points in all four quadrants of the coordinate plane (6.NS.6b, 6.NS.6c, 6.NS.7, 6.NS.8, 6.G.3)

Assessment Items:

1. ES 2, Demonstrator 1, Standard (6.NS.1)

“One serving of rice is $\frac{2}{3}$ of a cup. I ate 1 cup of rice. How many servings of rice did I eat?”

To solve the problem, Tonya and Chrissy draw a diagram divided into three equal pieces, and shade two of those pieces.



Tonya says, “There is one $\frac{2}{3}$ -cup serving of rice in 1 cup, and there is $\frac{1}{3}$ cup of rice left over, so the answer should be $1 \frac{1}{3}$.”

Chrissy says, “The answer is $\frac{3}{2}$ or $1 \frac{1}{2}$ ”

Who is right? Is the answer $1 \frac{1}{3}$ or $1 \frac{1}{2}$? Explain your reasoning using the diagram.

2. ES 2, Demonstrator 4, Standard (6.NS.7)

Order the lists of integers in order from greatest to least? -2, |7|, 1, -11, 8, |-2|

3. ES 2, Demonstrator 4, Standard (6.NS.8)

What is the distance between (-5,2) and (9,2) on a coordinate plane? Show your work.

4. ES 2, Demonstrator 1, Standard (6.NS.1)

Julie fed the horses $8\frac{1}{3}$ bales of hay over the course of 5 days.

How many bales of hay did she feed the horses each day?

5. ES 2, Demonstrator 1, Standard (6.NS.1)

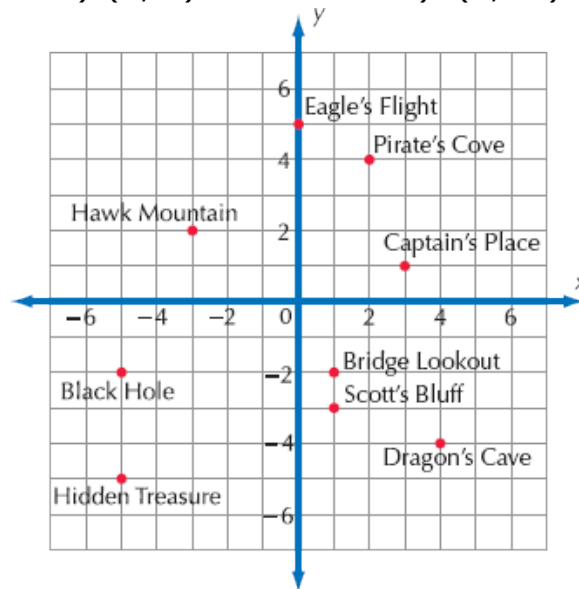
You have $\frac{3}{4}$ cups of butter. A cookie recipe calls for $\frac{1}{8}$ cup of butter.

How many batches of cookies can you make with what you have?

6. ES 2, Demonstrator 4, Standard (6.NS.8)

A) Mark drew a treasure map on a coordinate grid. What are the coordinates of Captain's Place?

- a) $(-1, 3)$ c) $(1, 3)$
b) $(3, 1)$ d) $(1, -3)$



B) What's the unit distance between *Black Hole* and *Bridge Lookout*?

7. ES 2, Demonstrator 2, Standard (6.NS.3, 6.RP.3c)

A dinner costs \$4.99. Joanie plans to tip the waitress 15%.
How much is the tip?

8. ES 2, Demonstrator 2, Standard (6.NS.2)

A school district sent a total of 924 science books to its fifth-grade classes.
Each fifth-grade class received 28 books. How many fifth-grade classes
are in the district?

9. ES 2, Demonstrator 2, Standard (6.NS.4)

Joshua is making bouquets of flowers to sell at the school carnival. The list below shows the number of each type of flower he is using to make the bouquets.

- 32 roses
 - 48 tulips
 - 64 carnations
- a) What is the maximum number of bouquets Joshua can make so that each bouquet has the same number of each flower and no flower is left over?
- b) If Joshua makes the maximum number of bouquets, how many roses, tulips, and carnations will be in each bouquet?

10. ES 2, Demonstrator 3, Standard (6.NS.5)

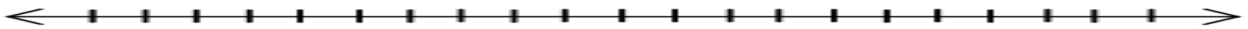
Positive and negative integers can be used to represent different scenarios.

- a) Use an integer to represent 25 feet below sea level.
- b) Use an integer to represent 25 feet above sea level.
- c) What would 0 (zero) represent in the scenario above?

11. ES 2, Demonstrator 3 and 4, Standard (6.NS.6)

- a) What is the opposite of $2\frac{1}{2}$? Explain your answer.
- b) Arrange the following numbers **and their opposites** on the number line below.

4.5, -5, $2\frac{1}{2}$, 0, 6



- c) Which of the numbers in part “b” does not have an opposite and why?

12. ES 2, Demonstrator 1, Standard (6.NS.1)

Solve $3 \div \frac{2}{5}$ or in other words, “how many $\frac{2}{5}$ are in 3” Show your work by writing an equation and drawing a visual model.

Students understand that a division problem such as $3 \div \frac{2}{5}$ is asking, “how many $\frac{2}{5}$ are in 3?” One possible visual model would begin with three wholes and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of $\frac{1}{2}$.

Therefore, $3 \div \frac{2}{5} = 7\frac{1}{2}$, meaning there are $7\frac{1}{2}$ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves.



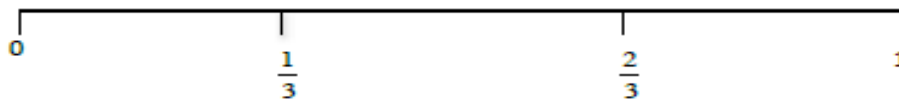
This section represents one-half of two-fifths

13. ES 2, Demonstrator 1, Standard (6.NS.1)

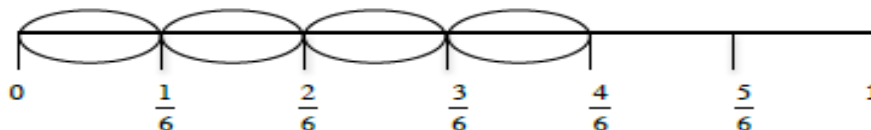
Susan has $\frac{2}{3}$ of an hour to make cards, it takes her about $\frac{1}{6}$ of an hour to make each card. How many will she be able to make? Model using a number line.

This problem can be modeled using a number line.

a. Start with a number line divided into thirds.



b. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.



c. Each circled part represents $\frac{1}{6}$. There are four sixths in two-thirds; therefore, Susan can make 4 cards.

14. ES 2, Demonstrator 4, Standard (6.NS.4)

Ms. Spain and Mr. France have donated a total of 90 hot dogs and 72 bags of chips for the class picnic. Each student will receive the same amount of refreshments. All refreshments must be used.

- a. What is the greatest number of students that can attend the picnic?
- b. How many bags of chips will each student receive?
- c. How many hotdogs will each student receive?

Solution:

- a. Eighteen (18) is the greatest number of students that can attend the picnic (GCF).
- b. Each student would receive 4 bags of chips.
- c. Each student would receive 5 hot dogs.

15. ES 2, Demonstrator 2, Standard (6.NS.4)

The elementary school lunch menu repeats every 20 days; the middle school lunch menu repeats every 15 days.

Both schools are serving pizza today. In how many days will both schools serve pizza again?

16. ES 2, Demonstrator 3, Standard (6.NS.6a)

What is the opposite of $2\frac{1}{2}$? Explain your answer.

Solution: $-2\frac{1}{2}$ because it is the same distance from 0 on the opposite side

17. ES 2, Demonstrator 4, Standard (6.NS.6b)

Graph the following points in the correct quadrant of the coordinate plane. If the point is reflected across the x-axis, what are the coordinates of the reflected points? What similarities are between coordinates of the original point and the reflected point?

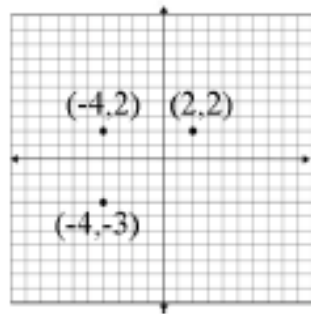
$(\frac{1}{2}, -3\frac{1}{2})$ $(-\frac{1}{2}, -3)$ $(0.25, 0.75)$

solution: $(\frac{1}{2}, 3\frac{1}{2})$ $(-\frac{1}{2}, 3)$ $(0.25, -0.75)$ the x coordinates remain the same while the y coordinates become the opposite

18. ES 2, Demonstrator 4, Standard (6.G.3)

The points on the coordinate plane below are the three vertices of a rectangle.

- What are the coordinates of the fourth vertex? How do you know?
- What are the length and width of the rectangle?
- Find the area **and** the perimeter of the rectangle.



Solution:

To determine the distance along the x-axis between the point $(-4, 2)$ and $(2, 2)$ a student must recognize that -4 is $|-4|$ or 4 units to the left of 0 and 2 is $|2|$ or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, $|-4| + |2|$. The length is 6 and the width is 5. The fourth vertex would be $(2, -3)$. The area would be 5×6 or 30 units².

The perimeter would be $5 + 5 + 6 + 6$ or 22 units.

19. ES 2, Demonstrator 4, Standard (6.G.3)

On a map, the library is located at $(-2, 2)$, the city hall building is located at $(0, 2)$, and the high school is located at $(0, 0)$. Represent the locations as points on a coordinate grid with a unit of 1 mile.

- What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?
- What shape does connecting the three locations form?

The city council is planning to place a city park in this area.

- How large is the area of the planned park?

Solution:

- The distance from the library to city hall is 2 miles. The coordinates of these buildings have the same y-coordinate. The distance between the x-coordinates is 2 (from -2 to 0).
- The three locations form a right triangle. The area is 2 square miles.

20. ES 2, Demonstrator 3, Standard (6.NS.5)

Denver, Colorado is called “The Mile High City” because its elevation is 5280 feet above sea level. Someone tells you that the elevation of Death Valley, California is -282 feet.

- A. Is Death Valley located above or below sea level? Explain.
- B. How many feet higher is Denver than Death Valley?
- C. What would your elevation be if you were standing near the ocean?

Solution:

- A. Death Valley is located below sea level. We know this because its elevation is negative. Sea level is the base for measuring elevation. Sea level elevation is defined as 0 ft. All other elevations are measured from sea level. Those places on Earth that are above sea level have positive elevations, and those places on Earth that are below sea level have negative elevations. Thus, Death Valley, with an elevation of -282 feet, is located below sea level.
- B. To find out how much higher Denver is than Death Valley, we can reason as follows:

Death Valley is 282 feet below sea level. Denver is 5280 above sea level. So to go from Death Valley to Denver, you would go up 282 feet to get to sea level and then go up another 5280 feet to get to Denver for a total of $282+5280=5562$.feet. Thus, Denver, Colorado is 5562 feet higher than Death Valley, California.

C. If you were standing near the ocean, your elevation would be close to zero. Depending on how high or low the tide is and where exactly you are standing, your elevation could be as low as -50 feet (or as high as 50 feet) if you are at the edge of a very low tide (or a very high tide, respectively) at the Bay of Fundy

21. ES 2, Demonstrator 3, Standard (6.NS.5)

Connor has \$73 dollars in his wallet, but he owes his friend \$24. Write an expression that represents the amount of money Connor will have after he pays back his friend?

22. ES 2 Demonstrator 1, Standard (6.NS.1)

How many full $1\frac{1}{4}$ inch strips can be cut from a string that is $4\frac{3}{4}$ inches long?

Show your work using an equation and/or a visual model.

Grade 6

Enduring Skill 3: Write, interpret, and evaluate expressions and equations

Demonstrators and Related Standards:

1. Understand the use of variables in mathematical expressions (6.EE.2B, 6.EE.6)
2. Write expressions, equations, and inequalities that correspond to given situations, including the use of exponents (6.EE.1, 6.EE.5, 6.EE.2a, 6.EE.7, 6.EE.8)
3. Evaluate expressions, equations, inequalities, and formulas to solve problems (6.G.1, 6.G.2, 6.G.4, 6.EE.5, 6.EE.8, 6.EE.2c, 6.EE.7)
4. Understand that expressions in different forms can be equivalent (6.EE.4)
5. Use properties of operations to generate equivalent expressions (6.EE.3)
6. Construct and analyze tables of quantities that are equivalent ratios (6.EE.9)
7. Use equations to describe relationships between quantities (6.EE.4)

Assessment Items:

1. ES 3, Demonstrator 2, Standard (6.EE.2)

To compute the perimeter of a rectangle you add the length, l and width, w and double this sum.

- a. Write an expression for the perimeter of a rectangle.
- b. Use the expression to find the perimeter of a rectangle with a length of 30 cm and width of 75 cm.

2. ES 3, Demonstrator 4,7, Standard (6.EE.4)

Which pair of expressions is equivalent?

- A $4(6x)$ and $10x$
- B $4(6x)$ and $24x$
- C $4x + 6x$ and $10x^2$
- D $4x + 6x$ and $24x$

3. ES 3, Demonstrator 2, Standard (6.EE.1)

What is the value of the expression below?

$$3^4 + 9$$

4. ES 3, Demonstrator 6, Standard (6.EE.9)

Wool socks sell for \$6 per pair, and cotton socks sell for \$5 per pair. Yesterday a store sold 12 pairs of socks. If the sales total was \$64 how many pairs of *wool* socks were sold? Use the equation $6w + 5c = 64$.

5. ES 3, Demonstrator 6, Standard (6.EE.9)

A train was traveling at a constant speed. The table below shows the distance in miles the train traveled in the first 4 hours.

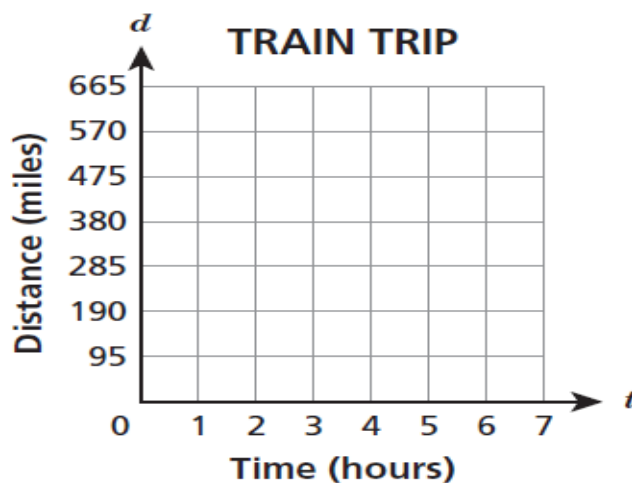
TRAIN TRIP

Time (hours)	Distance (miles)
1	95
2	190
3	285
4	380

Write an equation to represent the relationship between t , time and d , the total distance traveled by the train.

Answer: _____

On the grid below, draw a graph of the relationship between t and d for a trip that lasted 0 to 7 hours.



If the train is traveling non-stop, how many miles would it travel in $5 \frac{1}{2}$ hours?

Answer: _____

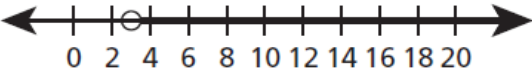
6. ES 3, Demonstrator 2 and 3, Standard (6.EE.7)

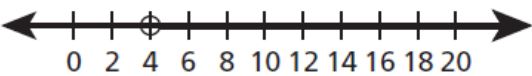
What is the solution to the equation $4w = \frac{2}{3}$

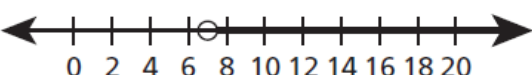
7. ES 3, Demonstrator 2, Standard (6.EE.8)

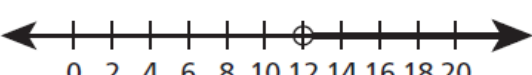
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A printer makes more than 3 copies of a book every hour. Which graph represents the number of books made in 4 hours?

A 

B 

C 

D 

8. ES 3, Demonstrator 2 and 3, Standard (6.EE.5)

The equation $0.44s = 11$ where s represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies used to determine the answer. Show that the solution is correct using substitution.

9. ES 3, Demonstrator 2 and 3, Standard (6.EE.5, 6.EE.8)

Twelve is less than 3 times another number can be shown by the inequality $12 \leq 3n$. What numbers could possibly make this a true statement?

10. ES 3, Demonstrator 1, Standard (6.EE.6)

Write an expression to represent Susan's age in three years, using s to represent her present age.

11. ES 3, Demonstrator 1, Standard (6.EE.6)

Write an expression to represent the number of wheels on any number of bicycles, b .

12. ES 3, Demonstrator 1, Standard (6.EE.6)

The skating rink charges \$100 to reserve the place and then \$5 per person. Write an expression to represent the cost for any number of people. Let n = the number of people.

13. ES 3, Demonstrator 1, Standard (6.EE.6)

An amusement park charges \$28 to enter and \$0.35 per ticket. Write an algebraic expression to represent the total amount spent.

14. ES 3, Demonstrator 1, Standard (6.EE.6)

Andrew has a summer job doing yard work. He is paid \$15 per hour and a \$20 bonus when he completes the yard. He was paid \$85 for completing one yard. Write an equation to represent the amount of money he earned.

15. ES 3, Demonstrator 1, Standard (6.EE.6)

Bill earned \$5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned.

16. ES 3, Demonstrator 4 and 7, Standard (6.EE.4)

Are the expressions equivalent? Explain your answer?

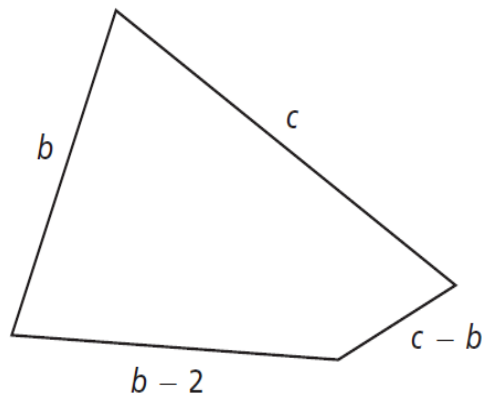
$$4m + 8 \quad 4(m+2) \quad 3m + 8 + m \quad 2 + 2m + m + 6 + m$$

Solution:

Expression	Simplifying the Expression	Explanation
$4m + 8$	$4m + 8$	Already in simplest form
$4(m+2)$	$4(m+2)$ $4m + 8$	Distributive property
$3m + 8 + m$	$3m + 8 + m$ $3m + m + 8$ $4m + 8$	Combined like terms
$2 + 2m + m + 6 + m$	$2m + m + m + 2 + 6$ $4m + 8$	Combined like terms Combined like terms

17. ES 3, Demonstrator 1, Standard (6.EE.6)

In the diagram of a quadrilateral below, the variables represent the lengths of the sides, in inches.



[not drawn to scale]

Write an expression using the variables b and c that could be used to find the perimeter of the quadrilateral.

Answer: _____

18. ES 3, Demonstrator 2 and 3, Standard (6.EE.5)

If $b=11$ and $c=16$, then what is the perimeter of the quadrilateral?
Show your work.

19. ES 3, Demonstrators 2 and 3, Standard (6.EE.7)

A fruit salad consists of blueberries, raspberries, grapes, and cherries. The fruit salad has a total of 280 pieces of fruit. There are twice as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries. How many cherries are there in the fruit salad?

20. ES 3, Demonstrators 2 and 3, Standard (6.EE.7)

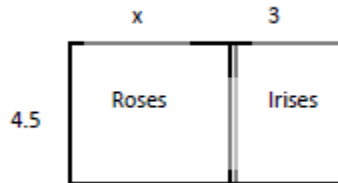
Represent the following expression algebraically:
A number, x , decreased by the sum of $2x$ and 5

- A. $(2x + 5) - x$
- B. $x - (2x + 5)$
- C. $x - 2x + 5$
- D. $(x + 2x) - 5$

Key: B

21. ES 3, Demonstrator 5, Standard (6.EE.3)

Given that the width is 4.5 units and the length can be represented by $x + 2$, the area of the flower can be expressed as $4.5(x + 3)$ or $4.5x + 13.5$.



Solve for x

22. ES 3, Demonstrator 5, Standard (6.EE.3)

Prove that $y + y + y = 3y$

Solution:

$$y + y + y$$

$$y \cdot 1 + y \cdot 1 + y \cdot 1 \text{ Multiplicative Identity}$$

$$y \cdot (1 + 1 + 1) \text{ Distributive Property}$$

$$y \cdot 3$$

$$3y \text{ Commutative Property}$$

23. ES 3, Demonstrators 2 and 3, Standard (6.EE.5)

Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?

24. ES 3, Demonstrator 5, Standard (6.EE.3)

Elk Street

The student government at the Elk Street Middle School is planning a fundraiser by holding three separate events: a school dance, a T-shirt/wrist band sale, and a walkathon. They are hoping to raise \$3,000. There are approximately 350 students in the whole middle school.

Part A

The T-shirts for the fundraiser will cost \$4 to make and there is a \$25 delivery fee to have them delivered. There are two different sized wristbands: a large one that will cost \$5, and a small one that will cost \$3 to make.

1) Write an expression to represent the student government's total cost of ordering t amount of T-shirts.

2a) If everyone who orders wristbands orders one small and one large wristband, write two equivalent expressions to represent the cost the student government will have to pay to have the wristbands made.

2b) Explain how you know that these two expressions are equivalent using mathematical terms such as product, quotient, factor, sum, and coefficient.

25. ES 3, Demonstrators 2, 3, and 6, Standard (6.EE.7, 6.EE.8, 6.EE.9)

Kayla's cell phone plan has a fixed fee of \$45 per month plus \$0.10 per text message. Kayla's mom said she would take away the phone if she spent more than \$60 on her phone service this month.

- Write an expression that can be used to represent Kayla's cell phone plan.
- Write an inequality to represent the number of text messages that Kayla can send or receive to stay within her budget and keep her phone.
- Kayla's mom says that Kayla can send 150 texts before going over the \$60. Is Kayla's mom correct? Justify your answer using mathematical reasoning, and show how many texts Kayla can send on the number line below.



26. ES 3, Demonstrators 2, 3, and 6, Standard (6.EE.7, 6.EE.8, 6.EE.9)

Last year, three 6th grade students, Aliya, Crystal, and Shamika, were selling balloons at the school's Winter Dance to collect money for a class trip to Spain.

Aliya sold two more balloons than Crystal, and Shamika sold twice as many balloons as Crystal. Let b represent the total number of balloons Crystal sold.

- Write an expression to represent the number of balloons Aliya sold.
- Write another expression to represent the number of balloons Shamika sold.
- If each balloon costs \$2 and altogether they made \$900, write an equation to represent the total sale.
- Solve the equation you wrote in part (c) to find the value of the variable, b . Show your work.
- How many balloons did each student sell? Show your work

27. ES 3, Demonstrator 3, Standard (6.G.1)

Find the area of a right triangle with a base length of three units, a height of four units, and a hypotenuse of 5.

Solution:

Students understand that the hypotenuse is the longest side of a right triangle. The base and height would form the 90 degree angle and would be used to find the area using: $A = \frac{1}{2} bh$

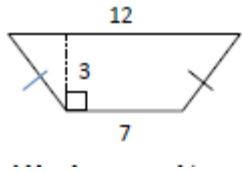
$$A = \frac{1}{2} (3 \text{ units})(4 \text{ units})$$

$$A = \frac{1}{2} 12 \text{ units}^2$$

$$A = 6 \text{ units}^2$$

28. ES 3, Demonstrator 3, Standard (6.G.1)

Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



Solution:

The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units².

The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there is a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be $\frac{1}{2}$ (2.5 units)(3 units) or 3.75 units²

2. Using this information, the area of the trapezoid would be:

21 units²

3.75 units²

+3.75 units²

28.5 units²

29. ES 3, Demonstrator 3, Standard (6.G.2)

A right rectangular prism has edges of $1\frac{1}{4}$ " , 1" and $1\frac{1}{2}$ ". How many cubes with side lengths of $\frac{1}{4}$ " would be needed to fill the prism? What is the volume of the prism?

Solution:

The number of $\frac{1}{4}$ " cubes can be found by recognizing the smaller cubes would be $\frac{1}{4}$ " on all edges, changing the dimensions to $\frac{5}{4}$ " , $\frac{4}{4}$ " and $\frac{6}{4}$ ". The number of one-fourth inch unit cubes making up the prism is 120 ($5 \times 4 \times 6$).

Each smaller cube has a volume of $\frac{1}{64}$ ($\frac{1}{4}$ " \times $\frac{1}{4}$ " \times $\frac{1}{4}$ "), meaning 64 small cubes would make up the unit cube.

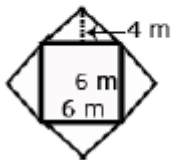
Therefore, the volume is $\frac{5}{4} \times \frac{6}{4} \times \frac{4}{4}$ or $\frac{120}{64}$ (120 smaller cubes with volumes of $\frac{1}{64}$ or $1\frac{56}{64}$ \rightarrow 1 unit cube with 56 smaller cubes with a volume of $\frac{1}{64}$).

30. ES 3, Demonstrator 3, Standard (6.G.4)

Find the surface area of the rectangular pyramid below using both diagrams below.



Bottom Diagram is a net of the top diagram.



31. ES 3, Demonstrator 4 and 7, Standard (6.EE.4)

Simplify by combining like terms

$$5q + 4q - 4q$$

- 5q
- 0
- 3q
- 16q

32. ES 3, Demonstrator 4,7, Standard (6.EE.4)

Write an expression equivalent to $2a + (b + 7c)$

33. ES 3, Demonstrator 5, Standard (6.EE.3)

Which expression is equivalent to $3(6m) + m$?

- A $19m$
- B $21m$
- C $7m + 3$
- D $18m + 6m^2$

Grade 6

Enduring Skill 4:

Develop understanding of statistical thinking

Demonstrators and Related Standards:

1. Understand that a statistical question has more than one answer and can be described by its center, spread, and overall shape (6.SP.1, 6.SP.2)
2. Recognize that a data distribution may not have a definite center and that different ways to measure center yield different values, for example the median is roughly the middle; where as, the mean is a balancing point of the data (6.SP.3)
3. Recognize that a measure of variability (interquartile range or mean absolute deviation) (6.SP.3, 6.SP.5c)
4. Describe, summarize, and create numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected (6.SP.4, 6.SP.5a, 6.SP.5b. 6.SP.5c, 6.SP.5d)

Assessment Items:

1. ES 4, Demonstrator 1, Standard (6.SP.1)

Zeke likes to collect buttons and he keeps them in a jar. Zeke can empty the buttons out of the jar, so he can see all of his buttons at once.



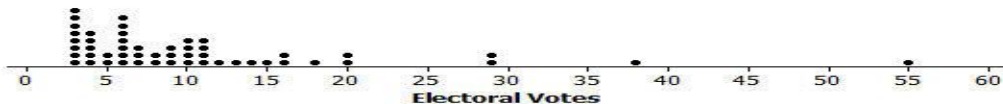
- a. Which of the following are statistical questions that someone could ask Zeke about his buttons? (A statistical question is one that anticipates an answer based on data that vary.) For each question, explain why it is or is not a statistical question.
- i. What is a typical number of holes for the buttons in the jar?
 - ii. How many buttons are in the jar?
 - iii. How large is the largest button in the jar?
 - iv. If Zeke grabbed a handful of buttons, what are the chances that all of the buttons in his hand are round?
 - v. What is a typical size for the buttons in the jar?
 - vi. How are these buttons distributed according to color?
- b. Write another statistical question related to Zeke's button collection.

2. ES 4, Demonstrators 1, 3, and 4, Standard (6.SP.2, 6.SP.5)

The following table shows the allocation of electoral votes for each state and the District of Columbia for the 2012, 2016, and 2020 presidential elections. (<http://www.archives.gov/federal-register/electoral-college/allocation.html> accessed September 4, 2012).

State	Electoral Votes	State	Electoral Votes	State	Electoral Votes
Alabama	9	Kentucky	8	North Dakota	3
Alaska	3	Louisiana	8	Ohio	18
Arizona	11	Maine	4	Oklahoma	7
Arkansas	6	Maryland	10	Oregon	7
California	55	Massachusetts	11	Pennsylvania	20
Colorado	9	Michigan	16	Rhode Island	4
Connecticut	7	Minnesota	10	South Carolina	9
Delaware	3	Mississippi	6	South Dakota	3
District of Columbia	3	Missouri	10	Tennessee	11
Florida	29	Montana	3	Texas	38
Georgia	16	Nebraska	5	Utah	6
Hawaii	4	Nevada	6	Vermont	3
Idaho	4	New Hampshire	4	Virginia	13
Illinois	20	New Jersey	14	Washington	12
Indiana	11	New Mexico	5	West Virginia	5
Iowa	6	New York	29	Wisconsin	10
Kansas	6	North Carolina	15	Wyoming	3

- Which state has the most electoral votes? How many votes does it have?
- Based on the given information, which state has the second highest population of qualified citizens?
- Here is a dot plot of the distribution.



- What measure of center (mean or median) would you recommend for describing this data set? Why did you choose this measure?
- Determine the value of the median for this data set (electoral votes).

3. ES 4, Demonstrator 3 and 4, Standard (6.SP.4, 6.SP.5c)

Each of the 20 students in Mr. Anderson's class timed how long it took them to solve a puzzle. Their times (in minutes) are listed below:

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Time (minutes)	3	5	4	6	4	8	5	4	9	5	3	4	7	5	8	6	3	6	5	7

- Display the data using a dot plot.
 - Find the mean and median of the data. Does it surprise you that the values of the mean and median are not equal? Explain why or why not.
 -
4. ES 4, Demonstrators 1,4, Standard (6.SP.2, 6.SP.4, 6.SP.5)

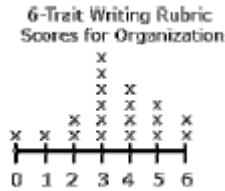
Below are the 25 birth weights, in ounces, of all the Labrador Retriever puppies born at Kingston Kennels in the last six months.

13, 14, 15, 15, 16, 16, 16, 16, 17, 17, 17, 17, 17, 17, 17, 18, 18, 18, 18, 18, 18, 18, 18, 19, 20

- Use an appropriate graph to summarize these birth weights.
- Describe the distribution of birth weights for puppies born at Kingston Kennels in the last six months. Be sure to describe shape, center and variability.
- What is a typical birth weight for puppies born at Kingston Kennels in the last six months? Explain why you chose this value.

5. ES 4, Demonstrator 1, Standard (6.SP. 2)

The dot plot shows the writing scores for a group of students on organization.



Describe the data.

6. ES 4, Demonstrators 3, 4, Standard (6.SP.5)

A researcher surveyed 50 people to find out how many home telephones they had.

The results are shown in the table

Home Telephone Survey

Number of People	Number of Home Telephones
1	0
2	1
19	2
16	3
10	4
2	5

- a. Which measure identifies the number of home telephones named by the greatest number of people?
- b. Calculate the mean, median, and range of the data. Which value best represents the data?

Solution:

The values range from 0 – 6. There is a peak at 3. The median is 3, which means 50% of the scores are greater than or equal to 3 and 50% are less than or equal to 3. The mean is 3.68. If all students scored the same, the score would be 3.68.

NOTE: Mode as a measure of center and range as a measure of variability are not addressed in the CCSS and as such are not a focus of instruction. These concepts can be introduced during instruction as needed.

7. ES 4, Demonstrator 2 and 3, Standard (6.SP.3)

Consider the data shown in the dot plot of the six trait scores for organization for a group of students.

- a. How many students are represented in the data set?
- b. What are the mean and median of the data set? What do these values mean? How do they compare?
- c. What is the range of the data? What does this value mean?



Solution:

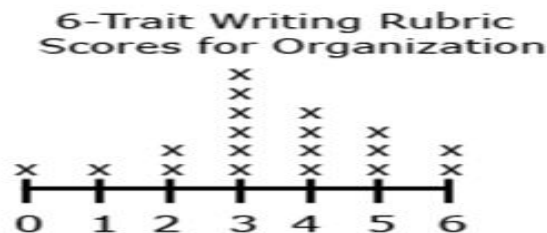
19 students are represented in the data set. • The mean of the data set is 3.5. The median is 3. The mean indicates that the values were equally distributed, all students would score a 3.5. The median indicates that 50% of the students scored a 3 or higher; 50% of the students scored a 3 or lower. • The range of the data is 6, indicating that the values vary 6 points between the lowest and highest scores.

8. ES 4, Demonstrator 4, Standard (6.SP.4)

Nineteen students completed a writing sample that was scored on organization. The scores for organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6.

- a. Create a data display.
- b. What are some observations that can be made from the data display?

Solution:



9. ES 4, Demonstrator 4, Standard (6.SP.4, 6.SP.5)

Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order.

- a. Create a data display.
- b. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

Solution: A histogram using 5 intervals (bins) 0-9, 10-19, ...30-39) to organize the data is displayed below.



Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.

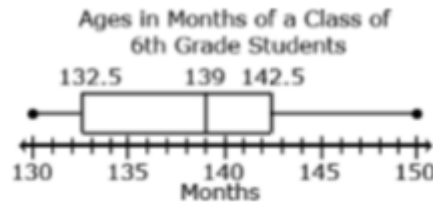
10. ES 4, Demonstrator 4, Standard (6.SP.4, 6.SP.5)

Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest.

- a. Create a data display.
- b. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

Solution: Five number summary Minimum – 130 months Quartile 1 (Q1) – $(132 + 133) \div 2 = 132.5$ months Median (Q2) – 139 months Quartile 3 (Q3) – $(142 + 143) \div 2 = 142.5$ months Maximum – 150 month



This box plot shows that • $\frac{1}{4}$ of the students in the class are from 130 to 132.5 months old • $\frac{1}{4}$ of the students in the class are from 142.5 months to 150 months old • $\frac{1}{2}$ of the class are from 132.5 to 142.5 months old • The median class age is 139 months.

11. ES 4, Demonstrator 4, Standard (6.SP.5)

Summarize numerical data sets in relation to their context, such as by:

- a. Reporting the number of observations.
- b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
- c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
- d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

12. ES 4, Demonstrators 3,4, Standard (6.SP.5c)

Susan has four 20-point projects for math class. Susan's scores on the first 3 projects are shown below:

- Project 1: 18
- Project 2: 15
- Project 3: 16
- Project 4: ??

What does she need to make on Project 4 so that the average for the four projects is 17? Explain your reasoning.

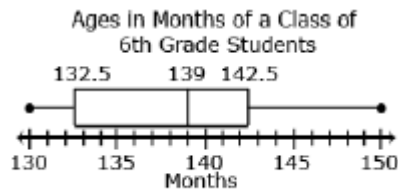
Solution:

One possible solution to is calculate the total number of points needed (17×4 or 68) to have an average of 17. She has earned 49 points on the first 3 projects, which means she needs to earn 19 points on Project 4 ($68 - 49 = 19$).

(Demonstrator 3, 4) (6.SP.5c)

13. ES 4, Demonstrator 3, 4, Standard (6.SP.5c)

What is the Interquartile Range (IQR) of the data below? What does this value represent in this situation?



Solution:

The first quartile is 132.5; the third quartile is 142.5. The IQR is 10 ($142.5 - 132.5$). This value indicates that the values of the middle 50% of the data vary by 10.

(Demonstrator 3,4) (6.SP.5c)

14. ES 4, Demonstrator 1, Standard (6.SP.2, 6.EE.8)

The following data set represents the size of 9 families:

3, 2, 4, 2, 9, 8, 2, 11, 4.

What is the MAD for this data set?

Solution:

The mean is 5. The MAD is the average variability of the data set. To find the MAD:

1. Find the deviation from the mean.
2. Find the absolute deviation for each of the values from step 1
3. Find the average of these absolute deviations.

The table below shows these calculations:

Data Value	Deviation from Mean	Absolute Deviation
3	-2	2
2	-3	3
4	-1	1
2	-3	3
9	4	4
8	3	3
2	-3	3
11	6	6
4	-1	1
MAD		$26/9 = 2.89$

This value indicates that on average family size varies 2.89 from the mean of 5. Students understand how the measures of center and measures of variability are represented by graphical displays. Students describe the context of the data, using the shape of the data and are able to use this information to determine an appropriate measure of center and measure of variability. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.

15. ES 4, Demonstrator 1, Standard (6.SP.2, 6.EE.8)

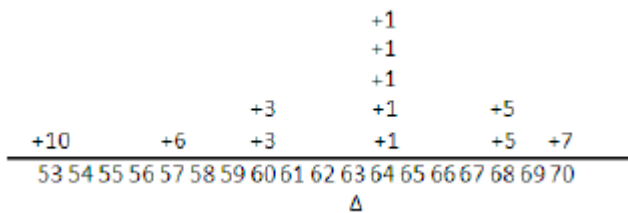
Here are the low temperatures (in Celsius) for one week in Juneau, Alaska:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
5	-1	-6	-2	3	7	0

- a) What is the shape of this distribution: skewed left, symmetric, or skewed right?
1. Imagine that someone you are speaking with is unfamiliar with these shape terms. Describe clearly and in the context of this data set what the shape description you have chosen means in terms of the distribution.
 2. Does the dot plot lead you to think that any states are outliers in terms of their number of electoral votes? Explain your reasoning, and if you do believe that there are outlier values, identify the corresponding states. Arrange them in order from coldest to warmest temperature.
 3. On a winter day, the low temperature in Anchorage was 23 degrees below zero (in degrees Celsius) and the low temperature in Minneapolis was 14 degrees below zero (in degrees Celsius). Sophia wrote, Minneapolis was colder because $-14 < -23$.
- b) Is Sophia correct? Explain your answer.
- c) The lowest temperature ever recorded on earth was -89°C in Antarctica. The average temperature on Mars is about -55°C . Which is warmer, the coldest temperature on earth or the average temperature on Mars? Write an inequality to support your answer.

16. ES 4, Demonstrator 2, Standard (6.SP.3)

From the following graph, what is the sum of the absolute deviations?



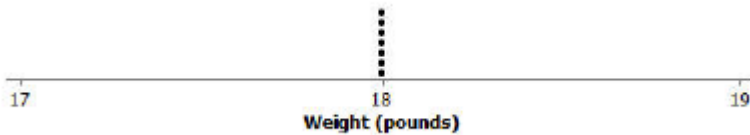
17. ES 4, Demonstrator 2, Standard (6.SP.3)

Suppose that the weights of seven middle-school students' backpacks are given below.

a. Fill in the following table.

Student	Alan	Beth	Char	Damon	Elisha	Fred	Georgia
Weight (lbs.)	18	18	18	18	18	18	18
Deviations	0	0	0	0	0	0	0
Absolute Deviations	0	0	0	0	0	0	0

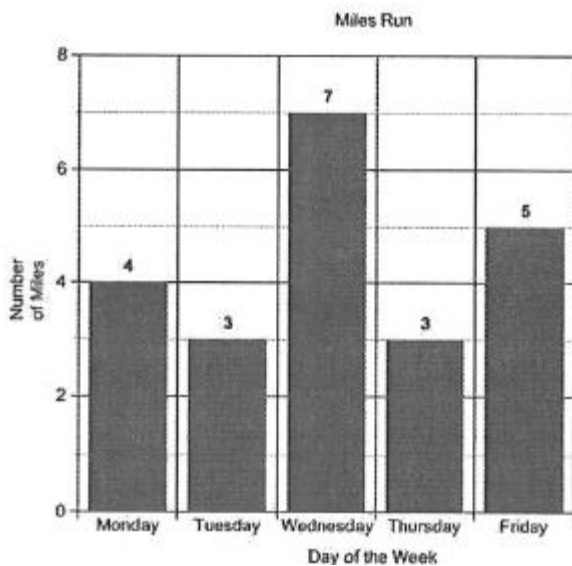
b. Draw a dot plot for these data and calculate the mean and MAD.



c. Describe this distribution of weights of backpacks by discussing the center, spread, and shape.

18. ES 4, Demonstrator 2, Standard (6.SP.3)

Find the mean, median, mode, and range of this data.



Resources:

<http://www.marsd.org/cms/lib7/NJ01000603/Centricity/Domain/387/Final%20Review%20Answer%20Key.pdf>

www.illustrativemathematics.com

www.engageny.com

www.ncpublicschools.org

www.ciits.com

www.parcc.com

www.mathshell.com

www.insidemathematics.com

www.reneeyates2math.com